Senior Division

Questions 1 to 10, 3 marks each
第 1-10 題，每題 3 分

1. What is the value of $(57 \times 346) + (43 \times 346)$?
請問 $(57 \times 346) + (43 \times 346)$ 的值是多少？

(A) 14 878 (B) 19 722 (C) 31 500 (D) 34 600 (E) 46 300

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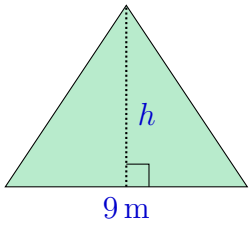
2. The area of the triangle shown is $36\,\text{m}^2$. What is the length h?
如圖所示三角形的面積為 $36\,\text{m}^2$。請問該三角形中 h 的長度為多少？



(A) 2 m (B) 4 m (C) 4.5 m (D) 8 m (E) 16 m

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3. What is the value of $\dfrac{1+\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}-\frac{1}{3}}$?
請問 $\dfrac{1+\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}-\frac{1}{3}}$ 的值為多少？

(A) -3 (B) -1 (C) 3 (D) 6 (E) 11

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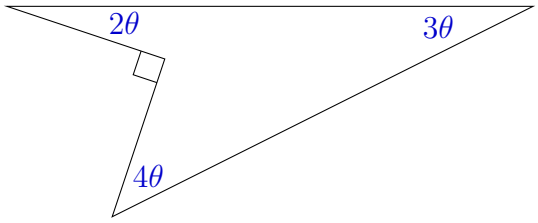
4. What is the value of θ in the diagram?
請問圖中 θ 的值為多少？



(A) 9° (B) 10° (C) 12° (D) 15° (E) 18°

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5. The largest integer n for which $0.6n$ is less than 17 is
若 $0.6n$ 小於 17，請問整數 n 的最大值是多少？

(A) 9 (B) 10 (C) 11 (D) 26 (E) 28

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6. If 2^n is a factor of 2024, then the largest possible value of n is
如果 2^n 是 2024 的一個因數，請問 n 的最大可能值是多少？

(A) 0 (B) 1 (C) 2 (D) 3 (E) 8

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7. A girl rides a scooter at a constant speed of 6 km/h. The number of metres she travels in one minute is
一位女孩按照 6 km/h 的速度勻速騎行滑板車。請問她 1 分鐘能騎多少 m？

(A) 20 (B) 100 (C) 200 (D) 400 (E) 600

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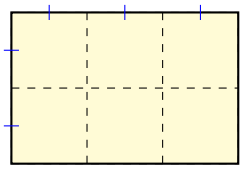
8. This rectangle has area 96 square centimetres. What is its perimeter, in centimetres?
已知該矩形的面積為 96 cm^2，請問其周長為多少 cm？



(A) 24 (B) 28 (C) 32 (D) 36 (E) 40

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9. Ell has many \$50 notes and no other currency, while Em has many \$20 notes and no other currency. What is the minimum number of notes that would need to be exchanged between the two if Ell would like to pay Em \$270?
Ell 有很多面額為 50 美元的紙幣；Em 有很多面額為 20 美元的紙幣。除此之外，這兩人都沒有其他貨幣。如果 Ell 需要向 Em 支付 \$270，那麼二者之間最少需要交換多少張紙幣？

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

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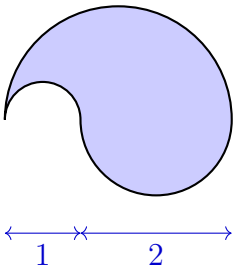
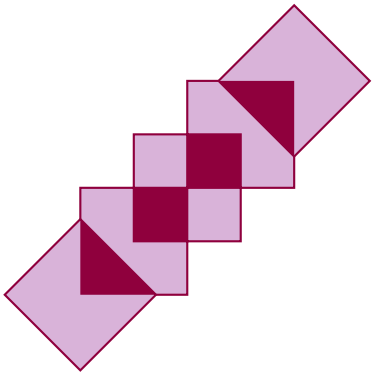
10. This teardrop shape is the logo for my new chat app. Its perimeter is composed of semicircular arcs. What is its perimeter?
如圖所示，這個淚滴形狀是我們全新推出的聊天軟體的圖示。其周長由半圓弧構成，請問這個形狀的周長是多少？



(A) π (B) 2π (C) 3π (D) 4π (E) 5π

Questions 11 to 20, 4 marks each
第 11-20 題，每題 4 分

11. This design is for part of a quilt. It is based on five overlapping squares of side 10 cm. The darker areas are two identical squares and two identical isosceles triangles. What is the total area of the design?
右圖所示的拼布設計由 5 個邊長為 10 cm 的正方形重疊構成。其中深色區域為 2 個相同正方形和 2 個相同等腰三角形。請問該設計的總面積是多少？



(A) 350 cm^2 (B) 400 cm^2 (C) 425 cm^2 (D) 450 cm^2 (E) 500 cm^2

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12. Michael has a pie to heat in the microwave oven. The instructions say to heat it for a number of minutes and seconds, longer than 4 minutes. He inputs the minutes and seconds as 3 digits. However the oven has been programmed to interpret the input as a 3-digit number of seconds. The pie heats for 60% longer than required. In seconds, how long should the pie have been heated?
Michael 有一個餡餅需要用微波爐加熱。說明書上寫明餡餅加熱所需時間為若干分和秒，並且這個時間超過 4 分鐘。他直接將說明書上的所需時間輸入為一個 3 位數，但微波爐的程式將他輸入的 3 位數的單位識別為秒。最終導致餡餅的實際加熱時間比所需時間長了 60%。請問加熱餡餅原本需要多少秒？

(A) 250 (B) 300 (C) 400 (D) 500 (E) 600

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13. Merle and Simon both collect toy cars. Before Merle's birthday, Simon had 50% more cars than Merle, but Merle got 20 cars for her birthday, so now she has 50% more cars than Simon. How many cars does Simon have?
Merle 和 Simon 都在收集玩具汽車。在 Merle 生日之前，Simon 擁有的玩具汽車數量比 Merle 多 50%。但是 Merle 過生日時又收到了 20 輛玩具汽車，她現在擁有的玩具汽車數量比 Simon 多 50%。請問 Simon 有多少輛玩具汽車？

(A) 12 (B) 15 (C) 20 (D) 24 (E) 30

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14. Which of the following is equal to $(2^7+3^8)^2$?
請問下列哪一項與 $(2^7+3^8)^2$ 的值相等？

(A) $4^7+6^8+9^8$
(B) $2^{14}+2^7\times 3^8+3^{16}$
(C) $2^{14}+3^{16}$
(D) $2^8+2\times 6^8+3^9$
(E) $2^{14}+6^{15}+3^{16}$

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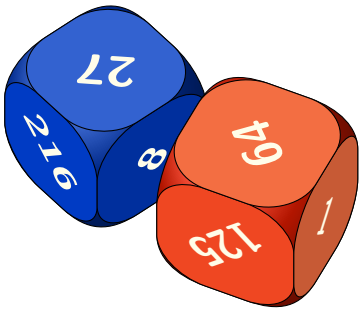
15. A *cubed cube* is a 6-sided dice with the numbers 1^3, 2^3, 3^3, 4^3, 5^3 and 6^3 on it. A red cubed cube and a blue cubed cube are rolled. What is the probability that the value on the red cube divided by the value on the blue cube is a whole number?
一個*三次方立方體* 是指一個各面數字分別為 1^3, 2^3, 3^3, 4^3, 5^3, 6^3 的 6 面骰子。分別投擲一個紅色三次方立方體和一個藍色三次方立方體。請問紅色三次方立方體所得數字除以藍色三次方立方體所得數字的結果為一個整數的概率是多少？



(A) $\frac{13}{36}$ (B) $\frac{5}{12}$ (C) $\frac{7}{18}$ (D) $\frac{4}{9}$ (E) $\frac{1}{3}$

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16. Depending on the value k, the equation $x^2+kx+2024=0$ can have 0, 1 or 2 integer solutions for x. For how many positive integer values of k does the equation have 2 integer solutions?
對於不同的 k 值，方程 $x^2+kx+2024=0$ 中 x 有 0, 1 或 2 個整數解。請問有多少個正整數 k 能夠使得上述方程有 2 個整數解？

(A) 3 (B) 6 (C) 8 (D) 14 (E) 16

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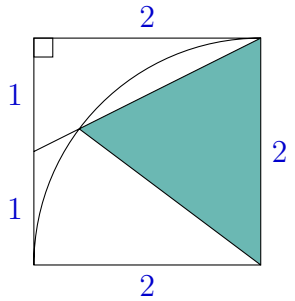
17. What is the shaded area in this diagram?
請問圖中陰影部分的面積是多少？



(A) $\frac{3}{2}$ (B) $\frac{8}{5}$ (C) $\frac{5}{3}$ (D) $\frac{7}{4}$ (E) 2

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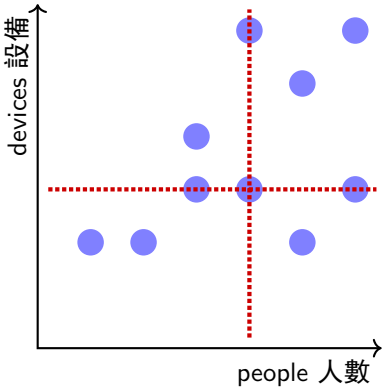
18. A researcher surveys 10 households and creates this scatterplot showing how many people and how many internet-connected devices are in each house.
一位研究人員對 10 個家庭進行了調研，並根據其調研結果繪製了如圖所示的散點圖，表示每個家庭中的人數與聯網設備數量之間的關係。



To help categorise her data, she adds a vertical line representing the median number of people and a horizontal line representing the median number of devices.
為了將數據分類，她在圖中加入一條垂線表示人數的中位數，又加入一條水平線表示設備數量的中位數。

One extra household is surveyed and the results are added to the scatterplot. What will happen to the median lines?
現在對另一個家庭進行調研，調研結果最終會表示在這個散點圖中。請問圖中兩條中位線將會有何變化？

(A) Both lines move. 兩條線都會移動。
(B) Neither line moves. 兩條線都不會移動。
(C) Only the vertical line moves. 只有垂線會移動。
(D) Only the horizontal line moves. 只有水平線會移動。
(E) It depends on the location of the new point. 取決於新加入點的位置。

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19. February 2024 had 5 Thursdays. When will February next have 5 Thursdays?
2024 年 2 月有 5 個週四。請問接下來哪一年的 2 月再次出現 5 個週四?

(A) 2028 (B) 2044 (C) 2048 (D) 2052 (E) 2080

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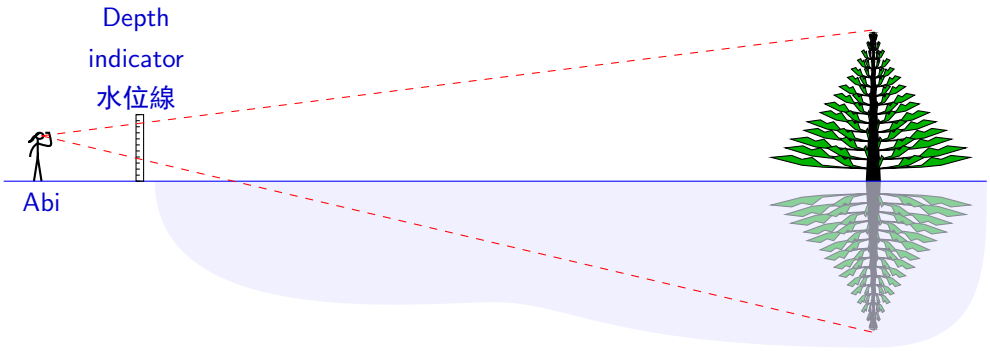
20. Abi is walking beside a lake. In the distance she can see a tree and its reflection on the surface of the lake. There is a depth indicator 5 metres away from her with its 0-metre mark at the lake surface level.

Abi 在湖邊散步。在某個距離，她剛好能看到一棵樹和樹在湖面的倒影。在距離她 5 m 處有一個水位線，湖面高度剛好達到 0 m 刻度處。



From Abi's point of view, the top of the tree is in line with the 1.8-metre mark on the depth indicator, while the reflection of the top of the tree is in line with the 1.1-metre mark. Abi's eyes are level with the 1.5-metre mark. How tall is the tree in metres?

從 Abi 的視角看，樹的頂部剛好與水位線 1.8 m 刻度處齊平，而樹的倒影的頂部與水位線 1.1 m 刻度處齊平。Abi 的眼睛與水位線 1.5 m 刻度處齊平。請問這棵樹高多少米？

(A) 9 (B) 9.5 (C) 10 (D) 10.5 (E) 11

Questions 21 to 25, 5 marks each
第 21-25 題, 每題 5 分

21. Marli has four fiction books and three non-fiction books on a shelf. How many ways are there to arrange these books so that no fiction books are next to one another?

Marli 的書架上有 4 本小說類書籍和 3 本非小說類書籍。請問有多少種排列方式可以使小說類書籍互不相鄰？

(A) 144 (B) 288 (C) 576 (D) 1440 (E) 5040

22. The equation $a(b^2 - c^2) = 2024$ has multiple solutions (a, b, c) where a, b and c are positive integers. How many such solutions are there?

方程 $a(b^2 - c^2) = 2024$ 有多個解 (a, b, c)，其中 a, b, c 都是正整數。請問該方程有多少個這樣的解？

(A) 4 (B) 9 (C) 12 (D) 13 (E) 17

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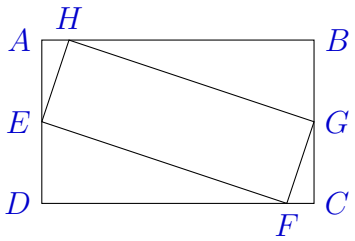
23. The rectangle $ABCD$ is twice as wide as high. The rectangle $EFGH$ is inscribed in $ABCD$ so that E is the midpoint of AD and G is the midpoint of BC, as shown. What is the size of angle $\angle DFE$?



矩形 $ABCD$ 的寬是高的兩倍。矩形 $EFGH$ 內接於矩形 $ABCD$，使得點 E 是 AD 邊中點，點 G 是 BC 邊中點，如圖所示。請問 $\angle DFE$ 為多少度？

(A) 15° (B) 18° (C) 20° (D) 22.5° (E) 30°

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24. Six maths students, Amy, Bao, Cecil, Daria, Emilia and Felipe are given 60 problems to complete over the weekend. Although none of them completes all 60, each completes at least 52 of the problems, and each one completes a different number of problems. Also the order from least to most problems is the same as the alphabetical order of their names.

Amy, Bao, Cecil, Daria, Emilia, Felipe 6 位學生需要在本週末完成 60 道題。雖然她們 6 個人都沒有將這 60 道題全部完成，但是每個人都至少完成了其中 52 道題且每個人完成的題目數量不同。同時，如果將這 6 位同學的題目完成數量從最少到最多進行排序，那麼所得順序剛好與這 6 位同學的名字按字母表順序進行排序的結果相同。

Finally, each of them completes k times as many problems on Saturday as they do on Sunday, where k is a different whole number from 1 to 6 for each of them. Which of them completed 3 times as many problems on Saturday as on Sunday?

最終，每個學生週六完成的題目數量都是其週日完成的題目數量的 k 倍。對於每個人來說，k 是 1 到 6 之間的一個不同整數。請問哪一位學生在週六完成的題目數量是其週日完成的題目數量的 3 倍？

(A) Amy (B) Bao (C) Cecil (D) Daria (E) Emilia

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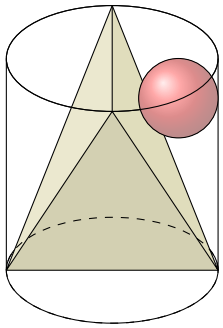
25. A solid regular tetrahedron of side length 2 is inscribed inside a hollow cylinder, so that two of its edges are diameters of the circular ends of the cylinder. What is the diameter of the largest sphere that can fit between the tetrahedron and the cylinder?



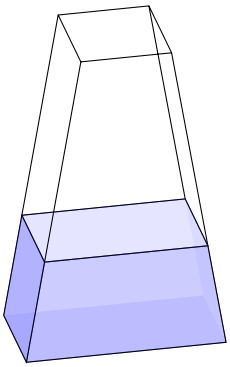
一個邊長為 2 的實心正四面體內接於一個空心圓柱，使得這個四面體的其中兩條邊是這個圓柱的兩個圓形底面的直徑。請問能放入四面體和圓柱之間的最大球體的直徑是多少？

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{3}}{3}$ (D) $1+\sqrt{2}-\sqrt{3}$ (E) $\sqrt{\frac{3}{2}}$

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.
第 26-30 題的答案為 0-999 之間的整數，請將答案填塗在答題卡對應區域。

Questions 26–30 are worth 6, 7, 8, 9 and 10 marks, respectively.
第 26-30 題分別為 6, 7, 8, 9, 10 分。

26. This vase is in the shape of a prism. The front and back faces are not rectangles, but the other four faces are. The front face has top and bottom sides parallel, and its left and right sides are of equal length.



如圖所示的花瓶是一個棱柱。這個花瓶除正面和背面外，其餘四面都是矩形。花瓶正面的上下兩條邊平行，左右兩條邊長度相同。

When I fill the vase to $\frac{1}{3}$ of its height, the vase is $\frac{1}{2}$ full.
當花瓶被填充至其高度的 $\frac{1}{3}$ 時，已經達到花瓶總容量的 $\frac{1}{2}$。

When I fill the vase to $\frac{1}{2}$ of its height, the vase is $\frac{m}{n}$ full. Here m and n are positive integers with no commmon factor other than 1. What is $m+n$?
當花瓶被填充至其高度的 $\frac{1}{2}$ 時，已經達到花瓶總容量的 $\frac{m}{n}$。其中 m 和 n 都是正整數且兩者除了 1 沒有其他公因數。請問 $m+n$ 的值為多少？

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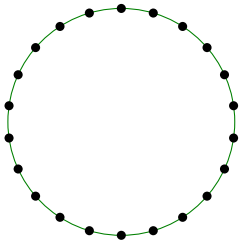
27. Twenty-two points are evenly spaced around a circle of radius 3. I draw line segments joining one of these points to all the other points. What is the sum of the squares of the lengths of these twenty-one line segments?



在一個半徑為 3 的圓上均勻分佈有 22 個點。用線段連接其中一個點與其他所有點。請問這 21 條線段的長度的平方和是多少？

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28. The sequence $a_1, a_2, a_3, \ldots$ starts with an integer a_1 and satisfies

數列 $a_1, a_2, a_3, \ldots$ 的首項 a_1 為整數，且對於每一個正整數 n，滿足：

$$a_{n+1} = \frac{3a_n - 1}{a_n + 1}$$

for every positive integer n. The 33rd term in the sequence is $a_{33} = \dfrac{2024}{1905}$.

數列中第 33 項為 $a_{33} = \dfrac{2024}{1905}$。

What is a_1, the first term of the sequence?

請問該數列的首項 a_1 的值是多少？

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29. Drew has 4 identical black socks, 4 identical red socks, 2 identical white socks, and 2 identical grey socks. He selects 5 socks at random from his sock drawer. The probability that he picks one pair of matching socks, but not a second matching pair, is $\dfrac{m}{n}$. Here m and n are positive integers with no common factor other than 1. What is $m + n$?

Drew 有 4 只相同的黑色襪子、4 只相同的紅色襪子、2 只相同的白色襪子和 2 只相同的灰色襪子。他從襪子抽屜中隨機抽出 5 只襪子。隨機抽取的襪子中只能匹配出一雙襪子的概率是 $\dfrac{m}{n}$。其中 m 和 n 都是正整數且除了 1 沒有其他公因數。請問 $m + n$ 的值為多少？

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30. The number $\sqrt[3]{4} + \sqrt[3]{6} + \sqrt[3]{9}$ is a root of a unique polynomial $p(x)$ with integer coefficients where the highest-power term is x^9, with coefficient 1. What is the absolute value of the coefficient of x^6 in $p(x)$?

存在唯一一個多項式 $p(x)$，各項係數均為整數，其中一個根為 $\sqrt[3]{4} + \sqrt[3]{6} + \sqrt[3]{9}$。這個多項式的最高次項是 x^9，其係數為 1。請問 $p(x)$ 中 x^6 的係數的絕對值是多少？
